

TRANSIENT STABILITY

Collected by: Prof. Shabram Montaser Kouhsari

TRANSIENT STABILITY

- In order to operate as an interconnected system all of the generators (and other synchronous machines) must remain in synchronism with one another
 - synchronism requires that (for two pole machines) the rotors turn at exactly the same speed at 3000 rev/min in 50 HZ system.
- Loss of synchronism results in a condition in which no net power can be transferred between the machines
- A system is said to be transiently unstable if following a disturbance one or more of the generators lose synchronism

TRANSIENT STABILITY

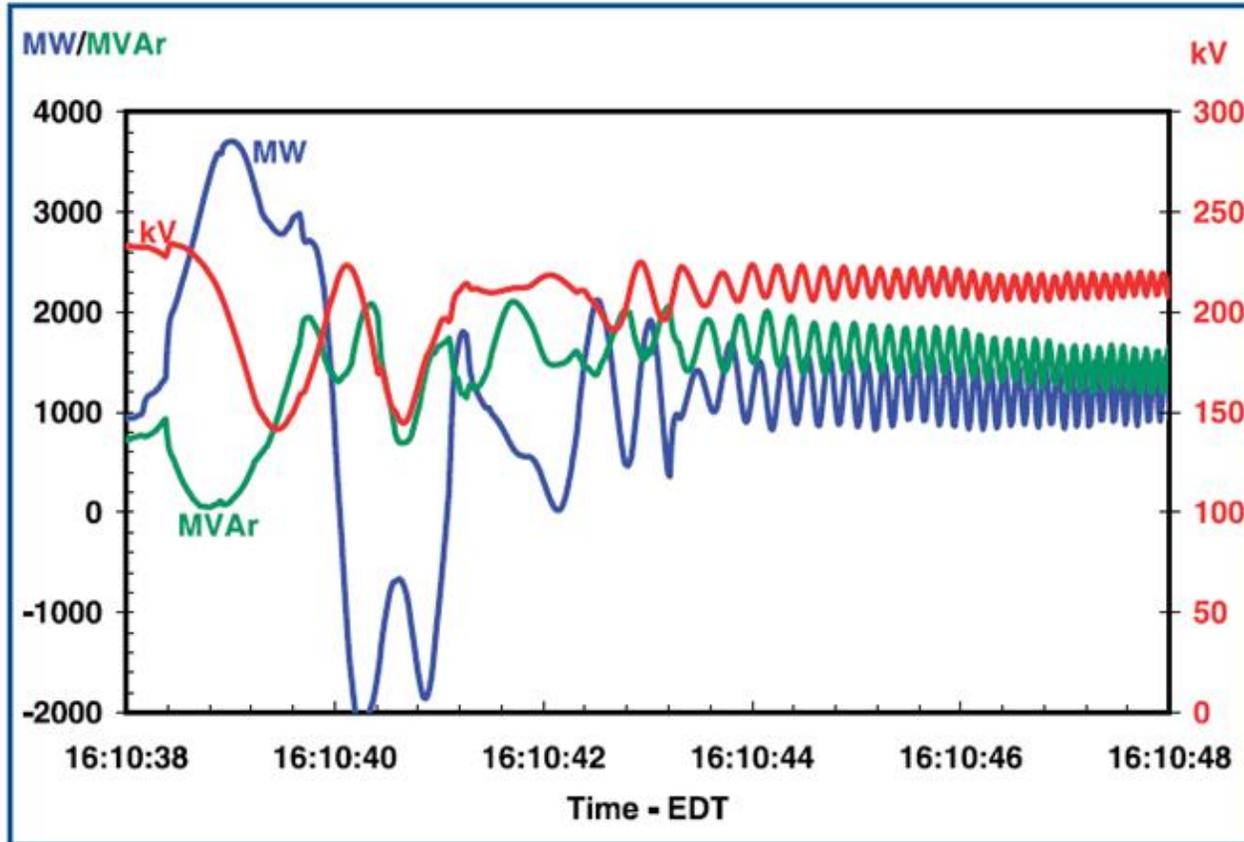


Figure shows MW and MVAR output of a generation station and the outgoing busbar voltage during a transient in an exemplified power system. It is started from an steady state condition (LOAD FLOW), and it is going to end to another steady state condition. The cause of the transient, might be due to line switching, or sudden interruption of a load, or trip of a generation station, or the most effective event in transient stability phenomena and that is the occurrence of a short circuit in the system, although short circuit in the network is cleared after a while (time required by the relays to sense the fault currents and orders to C.B. to open the faulted device + CB time to open the circuit).

During the transient time a generator or a group of generators will lose their synchronism. If they lose this **synchronism for a long time** (more than 1 second) a cascade condition might occur that may lead to the instability of the system. Indeed we have to trip the generators one by one and therefore a **black out** condition may occur.

TRANSIENT STABILITY

- If a black out or a brown out (a part of system is in black out condition) occurs, (indeed we have tripped and therefore lost all or some of our generators), the system restoration is a very difficult job and may take a couple of hours.
- The following are what we lose:
 - a) The utility do not sells electricity for a couple of hours
 - b) All industrial plants production will be affected
 - c) We will lose much of the energy produced due to releasing steam into the air
 - d) The people life will be affected
 - e) The safety of people will go on danger, the number of car accidents will rise
 - f) During restoration period, possible danger to our equipment might happen
- There are some other events might happen that may lead to the damage of the equipment. Following story has happened many times around the world:

TRANSIENT STABILITY

- In 2007 there was an explosion at the CWLP 86 MW Dallman 1 generator. The explosion was eventually determined to be caused by a sticky valve that prevented the cutoff of steam into the turbine when the generator went off line. So the generator turbine continued to accelerate up to over 6000 rpm (3600 normal in 60HZ system).
 - High speed caused parts of the generator to shoot out
 - Hydrogen escaped from the cooling system, and eventually escaped causing the explosion
 - Repairs took about 18 months, costing more than \$52 million

TRANSIENT STABILITY



TRANSIENT STABILITY



aser Kouhsari

TRANSIENT STABILITY

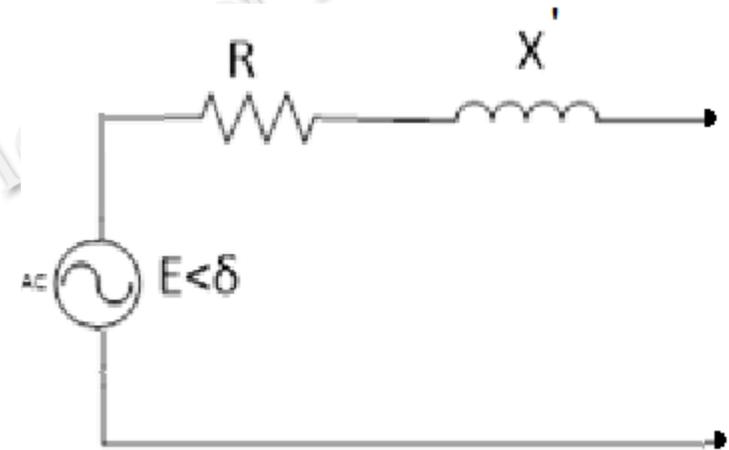
- Therefore we need to carefully study the transient stability phenomena, and prevent its occurrence.
- Although the following will mostly discuss about power system transmission and distribution those that are called the Grid, but the black out phenomena in industrial plants would also be a disaster and its occurrence cuts the production and causes a great loss to that industry.

TRANSIENT STABILITY

- As usual we need to develop models for all power system equipment involved in this phenomena. In order to study the transient response of a power system we have to have models for the generator valid during the transient time frame of several seconds following a system disturbance.
- For synchronous generators: it is required to develop both electrical and mechanical models of synchronous generator.

TRANSIENT STABILITY - Generator Electrical Model

- The simplest generator model, known as the classical model, treats the generator as a voltage source behind the direct-axis transient reactance; the voltage magnitude is fixed, but its angle changes according to the mechanical dynamics.
- The resistance can usually be ignored.



TRANSIENT STABILITY - Generator Mechanical Model

$$D\omega_m + J \frac{d\omega_m}{dt} = T_m - T_e \quad \text{Like:} \quad F = m \frac{dV}{dt} + kV$$

T_m = mechanical input torque (N.m)

J = moment of inertia of turbine & rotor ($kg \cdot m^2$)

ω_m = angular speed (rad/sec)

$D\omega_m$ = damping torque (N.m)

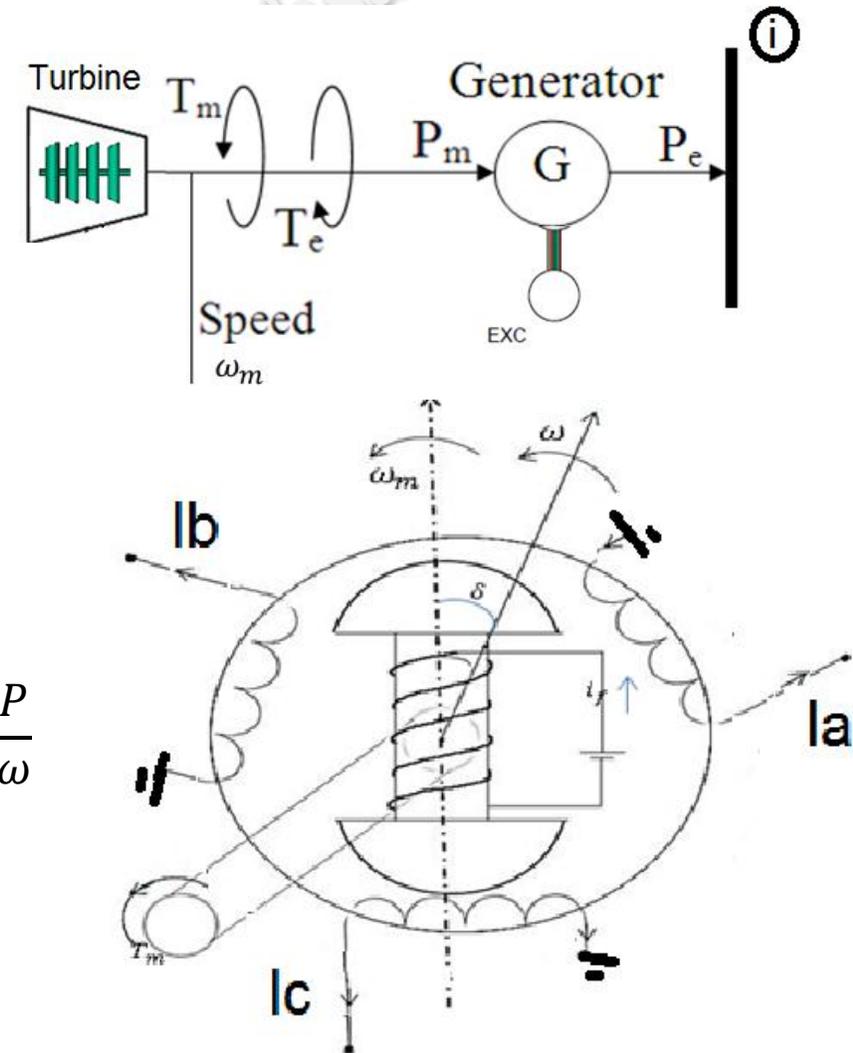
T_e = equivalent electrical torque (N.m)

In general torque can be related to power as:

$$T = \frac{\partial P}{\partial \omega} \xrightarrow{\text{Assume to be linear}} = \frac{\Delta P}{\Delta \omega} \xrightarrow{\text{Assume in power zero torque is zero and } \omega \text{ is zero}} = \frac{P}{\omega}$$

Therefore :

$$D\omega_m + J \frac{d\omega_m}{dt} = \frac{P_m}{\omega_m} - \frac{P_e}{\omega_m} \rightarrow D\omega_m^2 + J \omega_m \frac{d\omega_m}{dt} = (P_m - P_e)$$



TRANSIENT STABILITY

We assume that ω_m can not be changed too much, indeed the rotational speed changes more than 5 per cent [this is equivalent of 2.5 HZ in 50 HZ system and 150 rev/min (for two pole machines that its rotors turn at the speed of 3000 rev/min) will certainly unstable the generator. Therefore denoting ω_0 for synchronous speed or Rotating Observer speed we will have:

$$D\omega_0^2 + J\omega_0 \frac{d\omega_m}{dt} = (P_m - P_e)$$

Initially we will assume no damping i.e. $D=0$ then:

$$J\omega_0 \frac{d\omega_m}{dt} = (P_m - P_e)$$

Convert to p.u. by dividing by MVA base S_b and rearranging the equation:

TRANSIENT STABILITY

$$\frac{d\omega_m}{dt} = \frac{Sb}{J \omega_0} \left(\frac{P_m}{Sb} - \frac{P_e}{Sb} \right)$$

$$\frac{d\omega_m}{dt} = \frac{Sb}{J \omega_0} \left(\frac{P_m}{Sb} - \frac{P_e}{Sb} \right) = \frac{\frac{1}{2} \omega_0}{\frac{1}{2} J \omega_0^2 / Sb} \left(\frac{P_m}{Sb} - \frac{P_e}{Sb} \right)$$

Define: $H = \frac{1}{2} J \omega_0^2 / Sb$ as Inertia Constant where H is in KW.s/KVA or simply second and write the equation in p.u. terms:

$$\frac{d\omega_m}{dt} = \frac{\pi f_0}{H} (P_m - P_e)$$

TRANSIENT STABILITY

Knowing that: $\Delta\omega \triangleq \omega_m - \omega_0$ and ω_0 is constant we can then write:

$$\frac{d \Delta\omega}{dt} = \frac{\pi f_0}{H} (P_m - P_e)$$

On the other hand since R.O just sees angle and the following relationship exists between the angle and the speed difference:

$$\frac{d\delta}{dt} = \Delta\omega \quad \text{OR} \quad \Delta\omega = \frac{d\delta}{dt} \quad \text{Then:}$$

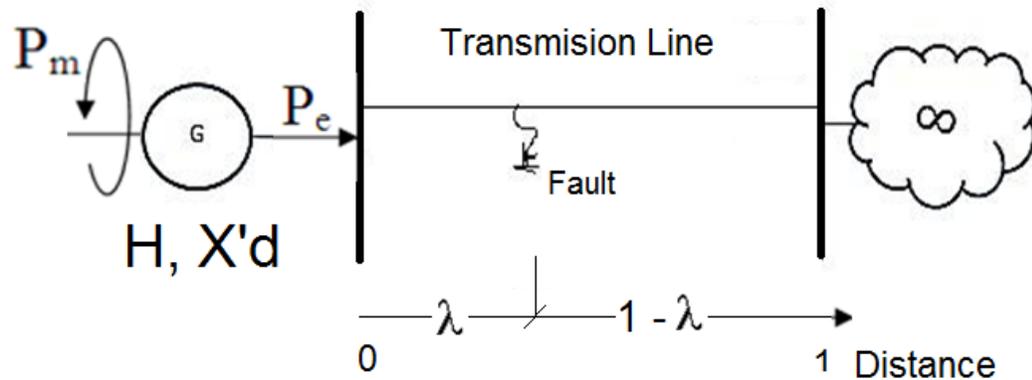
$$\frac{d^2\delta}{dt^2} = \frac{\pi f_0}{H} (P_m - P_e)$$

$$\left[\begin{array}{l} \frac{d \Delta\omega}{dt} = \frac{\pi f_0}{H} (P_m - P_e) \\ \frac{d\delta}{dt} = \Delta\omega \end{array} \right.$$

This equation (or these equations) is known as generator swing equation.

TRANSIENT STABILITY - Basics of stability problem

- To understand the transient stability problem we'll first consider the case of a single machine (synchronous generator) connected to a power system bus with a fixed voltage magnitude and angle (known as an infinite bus) through a transmission line with impedance jX



TRANSIENT STABILITY

- For transient stability analysis we need to consider three systems
 1. Pre-fault - before the fault occurs the system is assumed to be at an equilibrium point
 2. Due-fault - the fault changes the system equations, moving the system away from its equilibrium point
 3. Post-fault - after fault is cleared the system hopefully returns to a new operating point

Collected by: Prof. Dr. B. V. K. Rao

TRANSIENT STABILITY - Pre-fault

- In prefault condition, indeed the load flow condition governs the system. The system impedance diagram is as it is shown in the figure, and the below equations can be written:

1- The equation for P_e :

$$P_e = \frac{E V_\infty}{X'_d + X} \sin \delta = P_1 \sin \delta \quad \text{where} \quad P_1 = \frac{E V_\infty}{X'_d + X}$$

Assuming that before the fault generator is producing P_m p.u. output then:

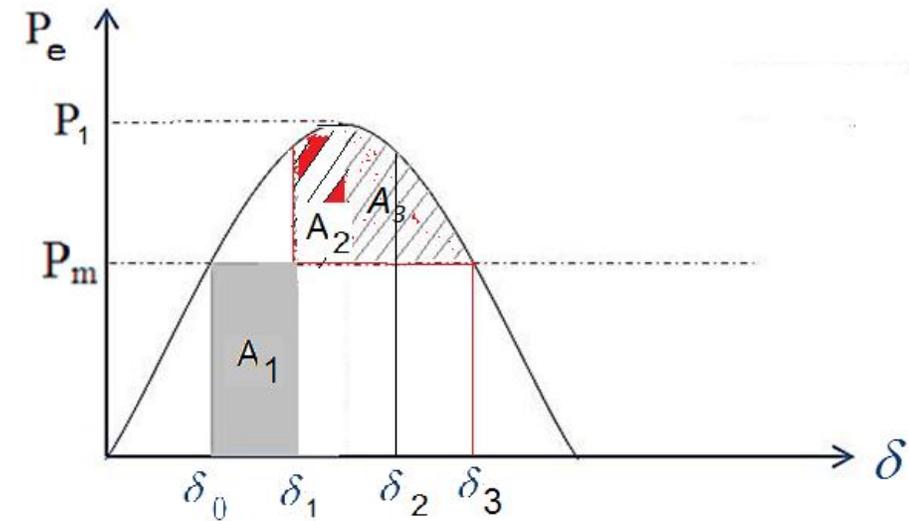
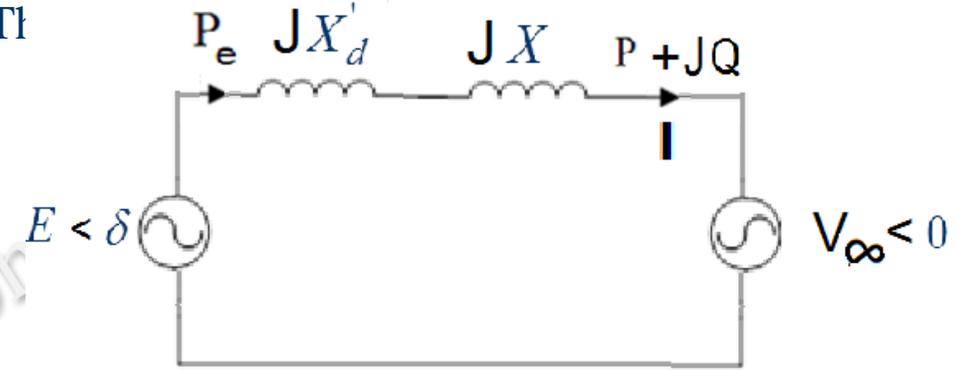
$$\delta = \delta_0 = \sin^{-1} \left(\frac{P_m}{P_1} \right)$$

2- E is constant and can be obtained from the load flow condition of the system:

$$E \angle \delta_0 = V_\infty \angle 0 + J(X'_d + X)I$$

where

$$I = \frac{P - JQ}{V_\infty \angle 0} \quad \text{and} \quad P = P_e \quad (\text{Lossless})$$



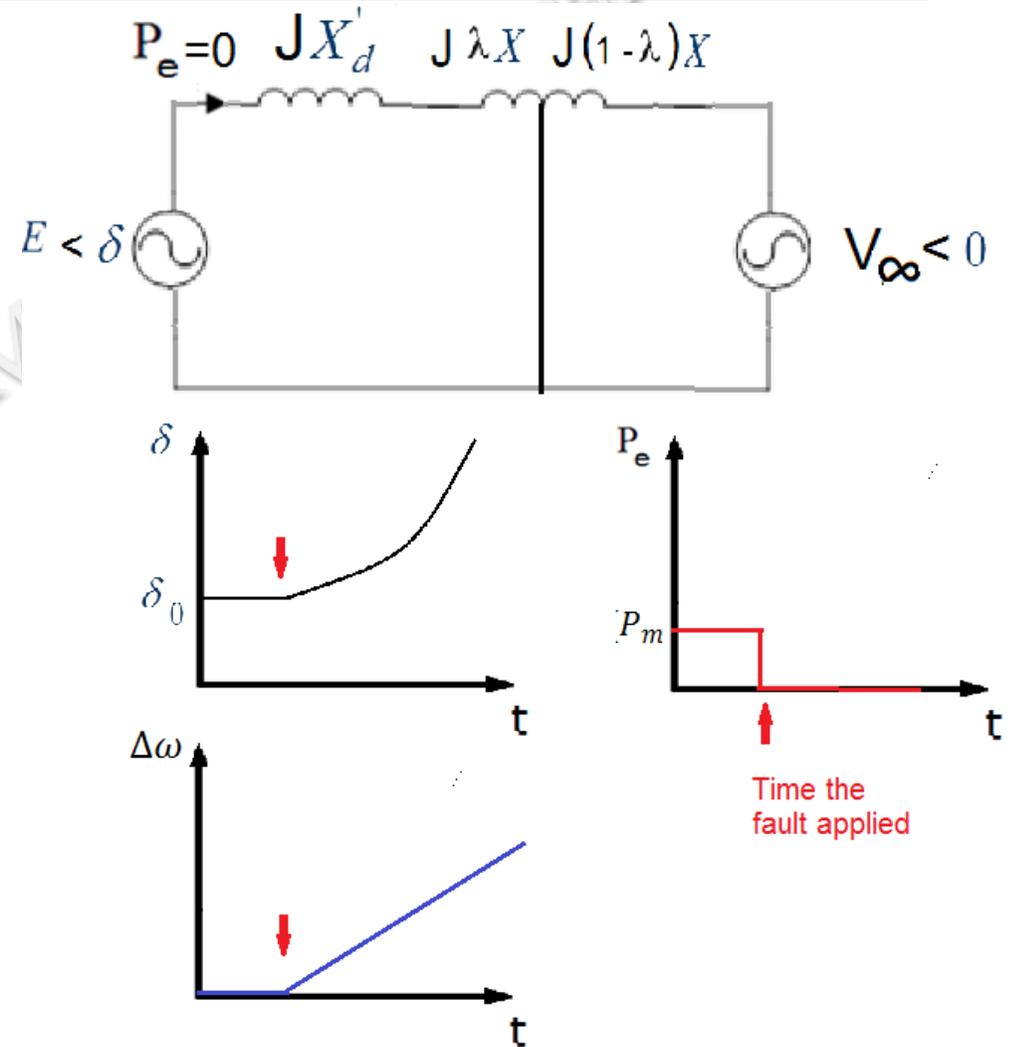
TRANSIENT STABILITY - Due-fault

- During the fault the active power output of generator will become zero (note that the current output of generator is too high but it is all reactive, since there is no resistance between the generator and the fault, and if it was its consumption is low, this can also be deduced from active power equation while V is zero in the fault point), but the P_m is constant since governor time constant is considered to be high, i.e. more than a second.

Therefore the generator speed and angle will increase according to the following formulas and as shown in the figures:

$$\frac{d \Delta \omega}{dt} = \frac{\pi f_0}{H} (P_m - 0) \quad \text{where } \Delta \omega_0 = 0 \quad \longrightarrow \quad \Delta \omega = \frac{\pi f_0}{H} P_m t + \Delta \omega_0$$

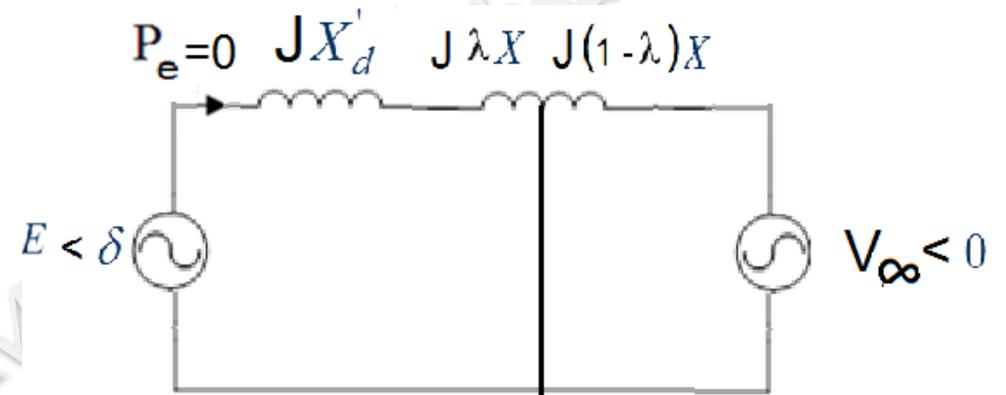
$$\frac{d\delta}{dt} = \Delta \omega \quad \longrightarrow \quad \delta = \frac{\pi f_0}{2H} P_m t^2 + \delta_0$$



TRANSIENT STABILITY - Due-fault

- If the fault sustains or removes by opening the circuit breakers (P_e is still zero), then the generator will lose its synchronism and it must be shut down in emergency condition after the frequency goes behind usually 52HZ. An over-frequency relay will detect this and order an appropriate action for emergency shut down of the plant. (shutting down a generator in emergency condition, means that open the field first and then send the steam into the air in order to prevent generator from speeding up. In normal shut down we just reduce the P_c gradually up to zero and then we open the circuit breaker).

Now consider that the fault will be removed by itself (might be due to blowing wind in the fault location), thereafter we will have the Post-fault condition as described below.



TRANSIENT STABILITY - Post-fault

- Now consider that the fault has removed after t_1 second, where, the angle is equal to δ_1 that can be obtained from the following:

$$\delta_1 = \frac{\pi f_0}{2H} P_m t_1^2 + \delta_0$$

Then P_e will go to :

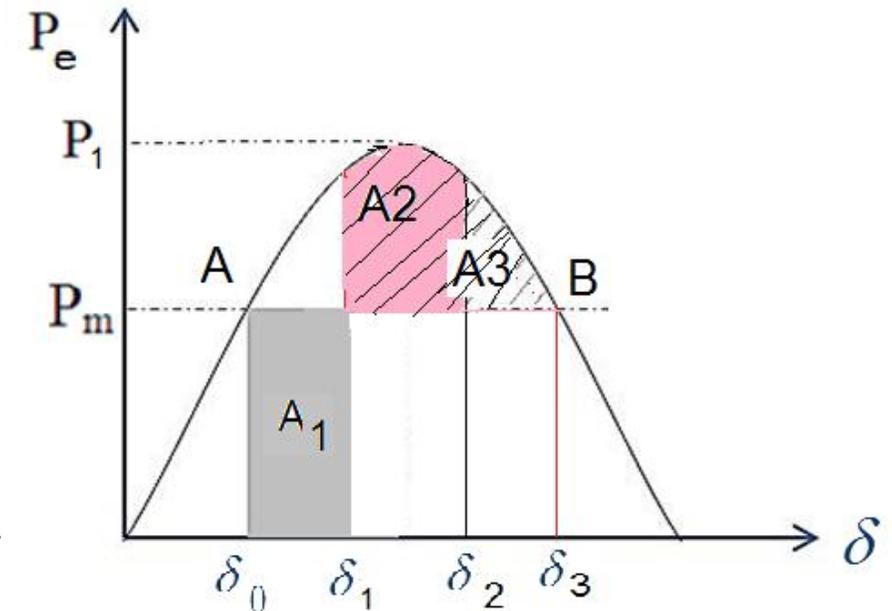
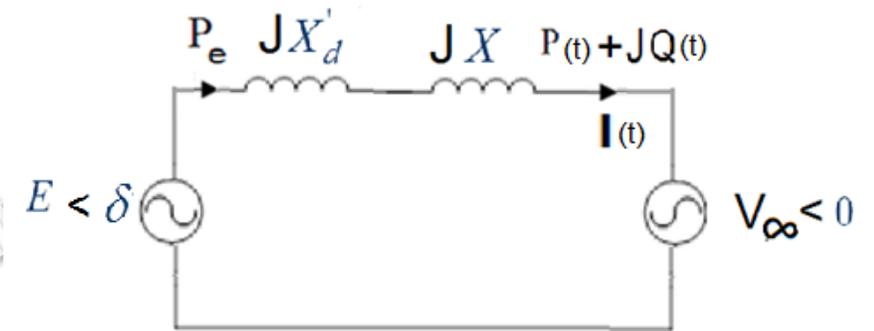
$$P_e = P_3 \sin \delta_1 \quad P_3 = P_1$$

(We have just defined P_3 to get a general expression with those in the next sections)

And together with δ continues to change according to the following equation:

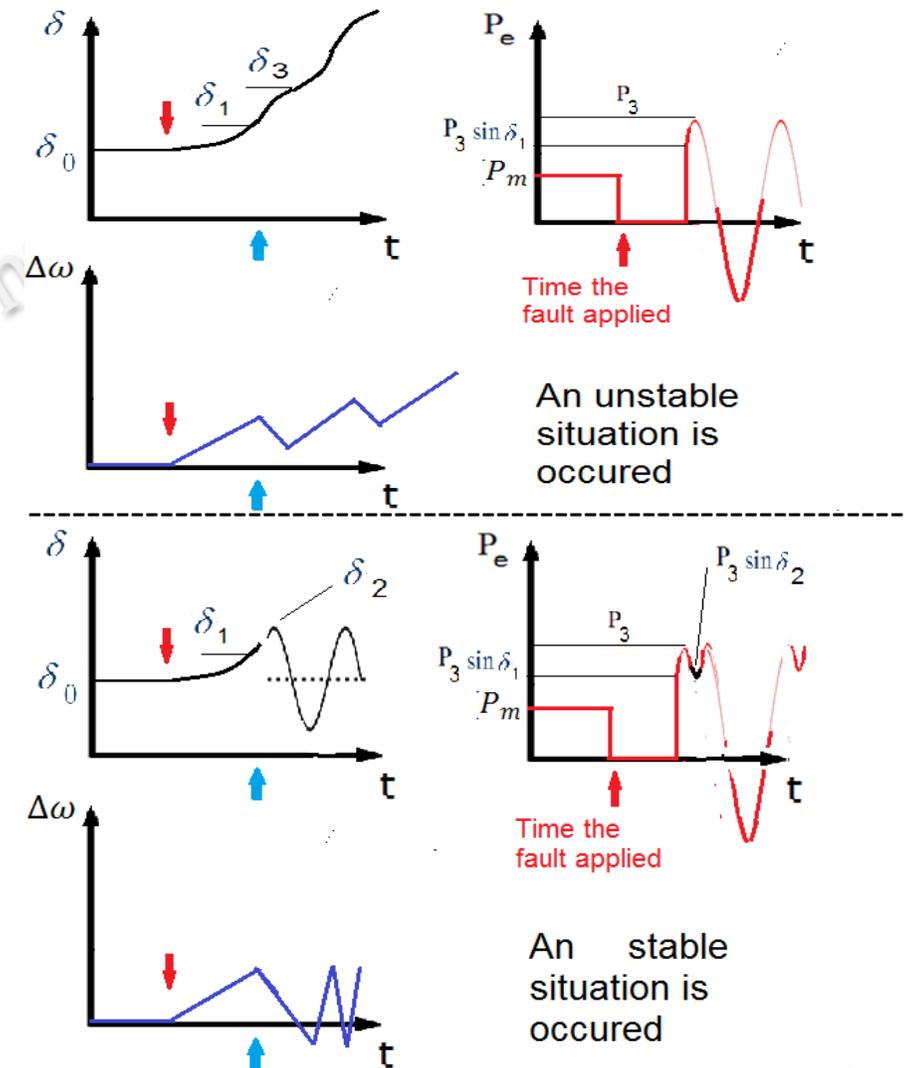
$$\frac{d^2 \delta}{dt^2} = \frac{\pi f_0}{H} (P_m - P_3 \sin \delta)$$

Unfortunately, there is no analytical solution for the above nonlinear differential equation in order to find how the angle will change and so to find the variation of power.



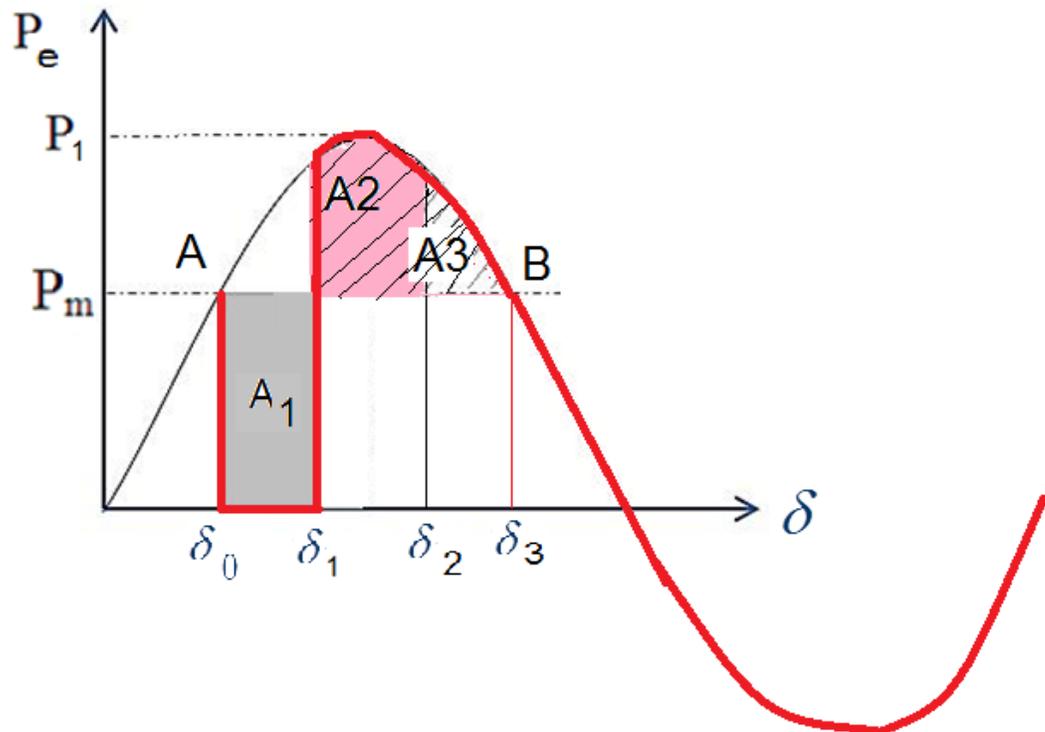
TRANSIENT STABILITY - Post-fault

- Although the P_e is more than the P_m at the post-fault condition assumed (the fault has been assumed to be removed at angle δ_1 which is less than the angle δ_3), but, we know that the angle will increase for a while since this is a second order differential equation, and the output do not follow the input. indeed it has memory what has gone to it before.
- Note that the speed has first order differential equation and it will be reduced instantly after the removal of the fault.
- The final trajectory of the angle has an oscillatory manner. The unstable figure show the situation when the angle has passed δ_3 during its oscillation but in stable figure the angle has not passed the δ_3 . When it passes δ_3 then the P_e (after that point) is less than P_m and therefore the angle will increase further and can not be stop by any controller or so. The point B sometimes called unstable point, while point A is a stable point for the operation of the generator.
- Note that in stable situation $\Delta\omega$ has passed zero.

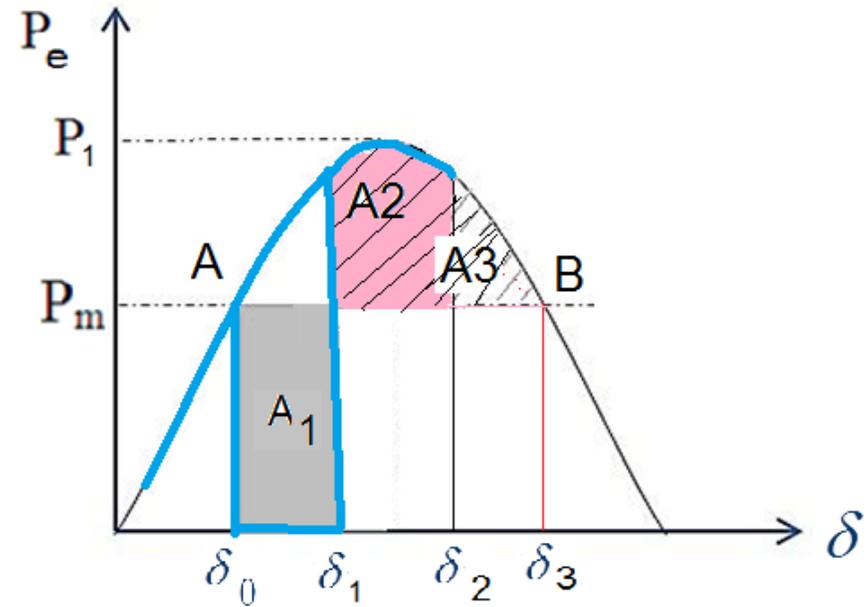


TRANSIENT STABILITY

The following figures show the trajectory of generator active power and its internal source angle in stable and unstable situation



Unstable situation



Stable situation

TRANSIENT STABILITY - Methods of solution

- There are two methods for solving the transient stability problem
 1. Numerical integration
 - this is by far the most common technique, particularly for large systems; during the fault and after the fault the power system differential equations are solved using numerical methods
 2. Direct or energy methods using Lyapunov function; for a two bus system (one synchronous generator and infinite busbar or one synchronous generator and a synchronous motor) this method is known as the equal area criteria
 - mostly used to provide an intuitive insight into the transient stability problem, therefore we start with equal area criteria.

TRANSIENT STABILITY - Equal Area Criteria

We need to solve this:

$$\frac{d^2 \delta}{dt^2} = \frac{\pi f_0}{H} (P_m - P_e)$$

From math's we have:

$$\frac{d}{dt} \left(\frac{d\delta}{dt} \right)^2 = 2 \frac{d^2 \delta}{dt^2} \frac{d\delta}{dt}$$

As in math's: $\frac{d}{dt} U^2 = 2U \frac{dU}{dt}$

Multiply the above equation to $2 \frac{d\delta}{dt}$

$$\frac{d}{dt} \left(\frac{d\delta}{dt} \right)^2 = \frac{2\pi f_0}{H} (P_m - P_e) \frac{d\delta}{dt}$$

OR

$$\frac{d}{dt} \Delta\omega^2 = \frac{2\pi f_0}{H} (P_m - P_e) \frac{d\delta}{dt}$$

TRANSIENT STABILITY - Equal Area Criteria

Omit dt from both side of the above equation and integrate that respect to δ

$$\Delta\omega^2 = \int_{\delta_0}^{\delta_2} 2 \frac{\pi f_0}{H} (P_m - P_e) d\delta$$

We know that if $\Delta\omega$ is zero, the system will be stable and the angle will go to δ_2 , therefore:

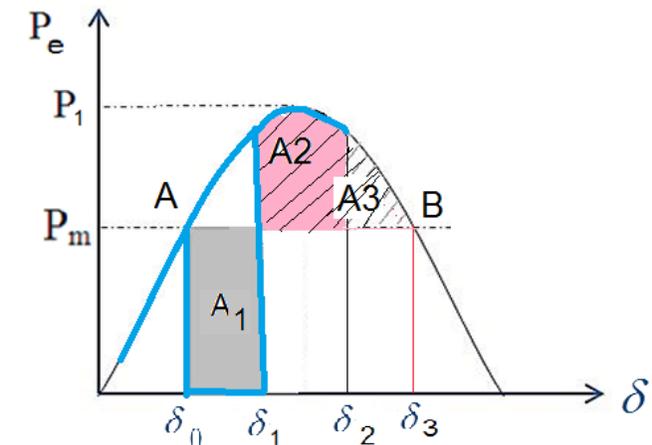
$$\int_{\delta_0}^{\delta_2} 2 \frac{\pi f_0}{H} (P_m - P_e) d\delta = 0$$

Since generally $\frac{2\pi f_0}{H}$ is constant, it can be omitted, writing the remaining as two separate integrals :

$$\int_{\delta_0}^{\delta_1} (P_m - P_e) d\delta + \int_{\delta_1}^{\delta_2} (P_m - P_e) d\delta = 0$$

Referring to power angle curve:

$$A_1 - A_2 = 0 \quad A_1 = A_2$$



TRANSIENT STABILITY - Equal Area Criteria

Where:

$$A1 = \int_{\delta_0}^{\delta_1} (P_m - P_e) d\delta$$

$$A2 = \int_{\delta_1}^{\delta_2} (P_e - P_m) d\delta$$

We do not know δ_2 in order to find A2, but the followings can be compared:

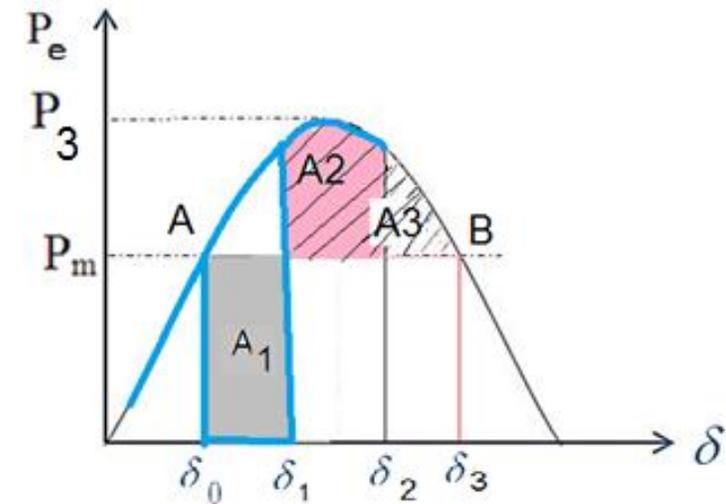
$A1 > A3$ The system is **unstable**

$A1 \leq A3$ The system is **stable**

$A3$
Where:

$$A3 = \int_{\delta_1}^{\delta_3} (P_e - P_m) d\delta$$

and we know $\delta_3 = \pi - \delta_0 = \pi - \sin^{-1}\left(\frac{P_m}{P_3}\right)$



TRANSIENT STABILITY - Equal Area Criteria

In the above equations all the angles are in radian. The critical angle δ_c as shown in the following figure can be found from:

$$A1 = A3 \quad \text{Where:}$$

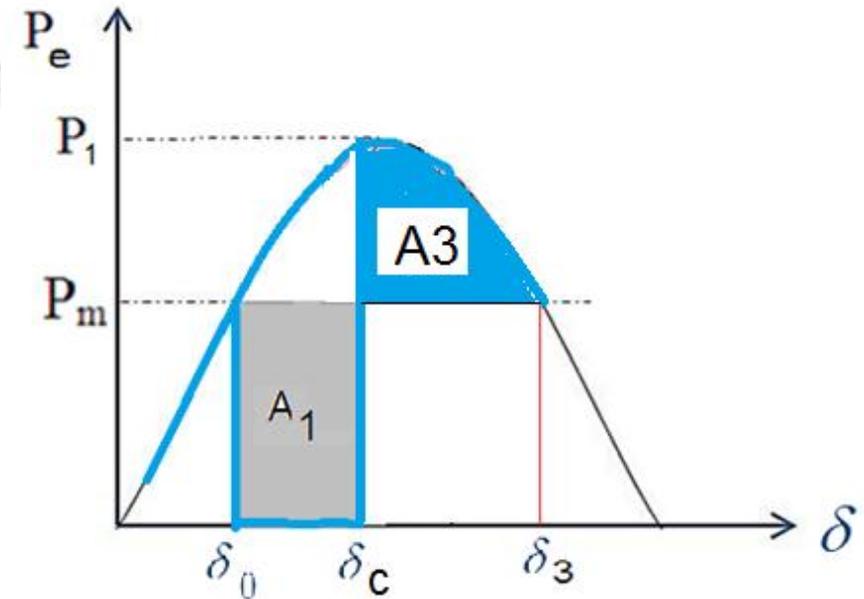
$$A1 = \int_{\delta_0}^{\delta_c} (P_m - P_e) d\delta$$

$$A3 = \int_{\delta_c}^{\delta_3} (P_e - P_m) d\delta$$

$$\cos \delta_c = \frac{P_m (\delta_3 - \delta_0) - P_2 \cos \delta_0 + P_3 \cos \delta_3}{P_3 - P_2}$$

And just when transmitting power during the fault is zero ($P_2 = 0$), the corresponding critical clearing time can be obtained from:

$$\delta_c = \frac{\pi f_0}{2H} P_m t_c^2 + \delta_0$$



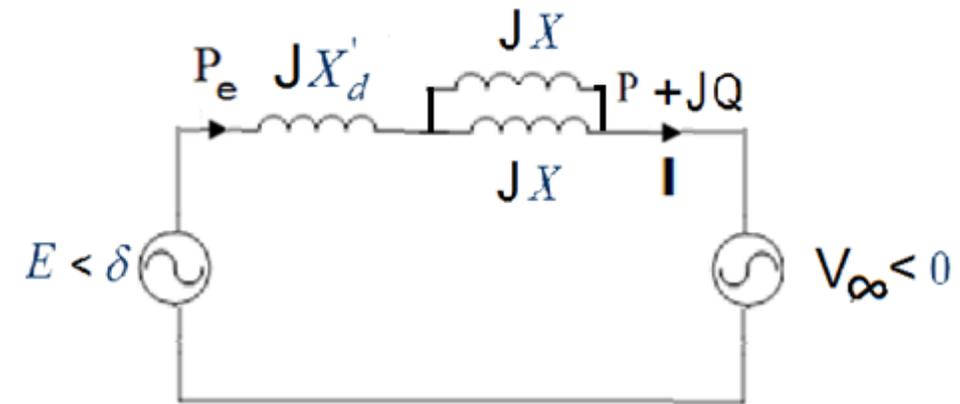
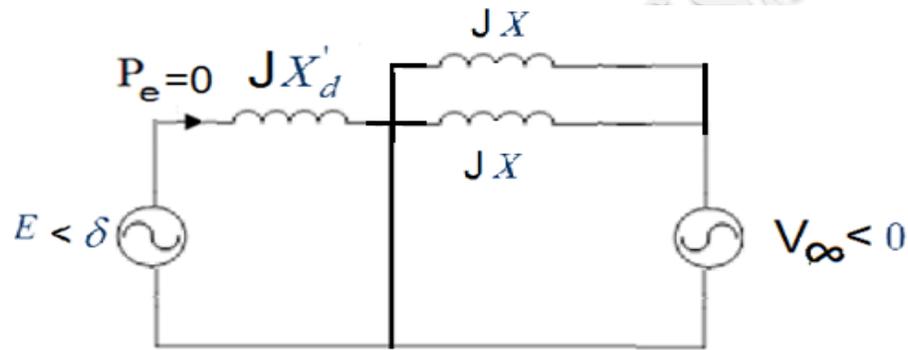
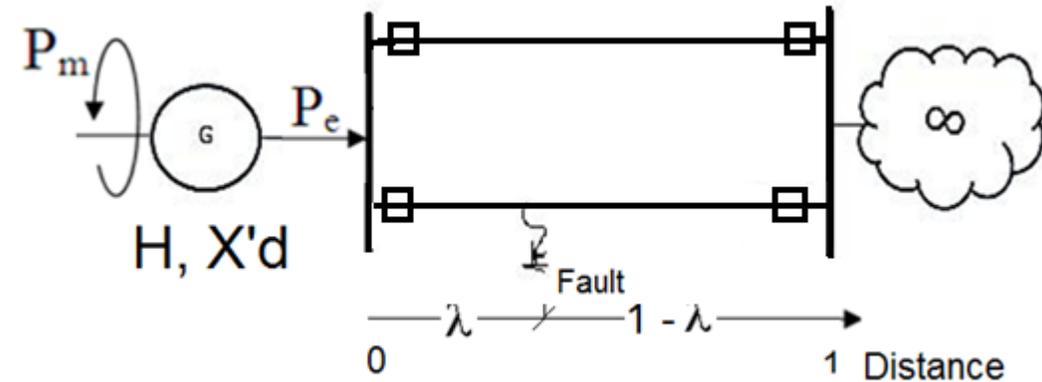
TRANSIENT STABILITY - Energy interpretation

- The above phenomena can be interpreted as that area A_1 corresponds to the kinetic energy increased during the fault and the area A_3 is the potential energy that opposes the kinetic energy the system gained during the fault.
- Such as interpretation will be used in Lyapunov function, which deals with multi-machine kinetic and potential energy of the system.
- Lyapunov function description is not covered in EE4004 syllabus and therefore, we just leave it for self study.

TRANSIENT STABILITY - Equal Area Criteria

More realistic example is shown in the below figure where two lines are connected between the generation station busbar and the infinite busbar, and a fault occurs in one of the lines.

At first we assume that the fault is near to the sending end or receiving end of the line, we this assumption we actually assumed that P_e is still zero during the fault, as can be concluded from the impedance diagram shown.



TRANSIENT STABILITY - Equal Area Criteria

At first we assume that the fault is near to the sending end or receiving end of the line ($\lambda = 0$ or $\lambda = 1$), with this assumption we actually assumed that P_e is still zero during the fault, as can be concluded from the impedance diagram shown.

1- Pre-fault:

$$P_e = \frac{E V_{\infty}}{X'_d + X/2} \sin \delta = P_1 \sin \delta$$

$$E \cos \delta_0 = V_{\infty} < 0 + J(X'_d + X/2)I$$

where
 $I = \frac{P - JQ}{V_{\infty} < 0}$ and $P = P_e$ (Lossless)

2- Due-fault:

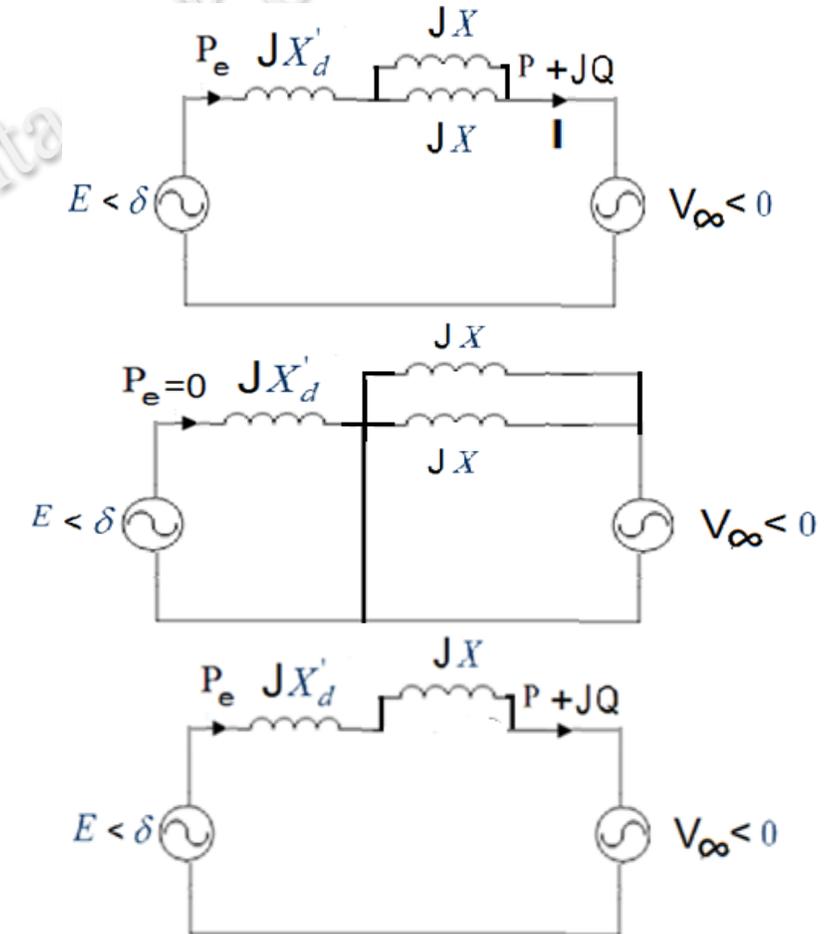
$$\frac{d \Delta \omega}{dt} = \frac{\pi f_0}{H} (P_m - 0) \quad \text{where } \Delta \omega_0 = 0 \quad \text{-----} \rightarrow \quad \Delta \omega = \frac{\pi f_0}{H} P_m t + \Delta \omega_0$$

$$\frac{d \delta}{dt} = \Delta \omega \quad \text{-----} \rightarrow \quad \delta = \frac{\pi f_0}{2H} P_m t^2 + \delta_0$$

3- Post-fault:

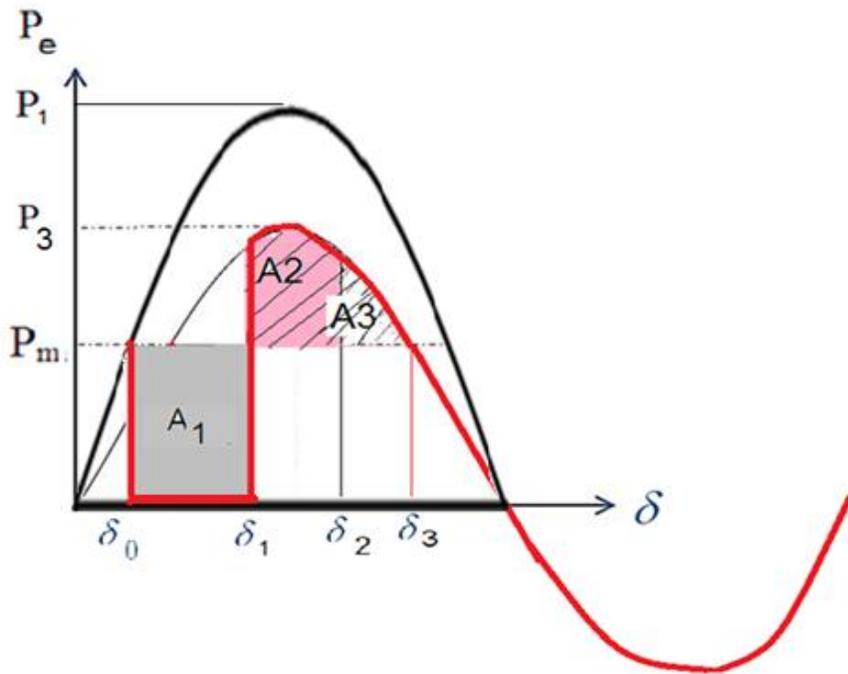
$$\frac{d^2 \delta}{dt^2} = \frac{\pi f_0}{H} (P_m - P_3 \sin \delta)$$

where $P_3 = \frac{E V_{\infty}}{X'_d + X}$

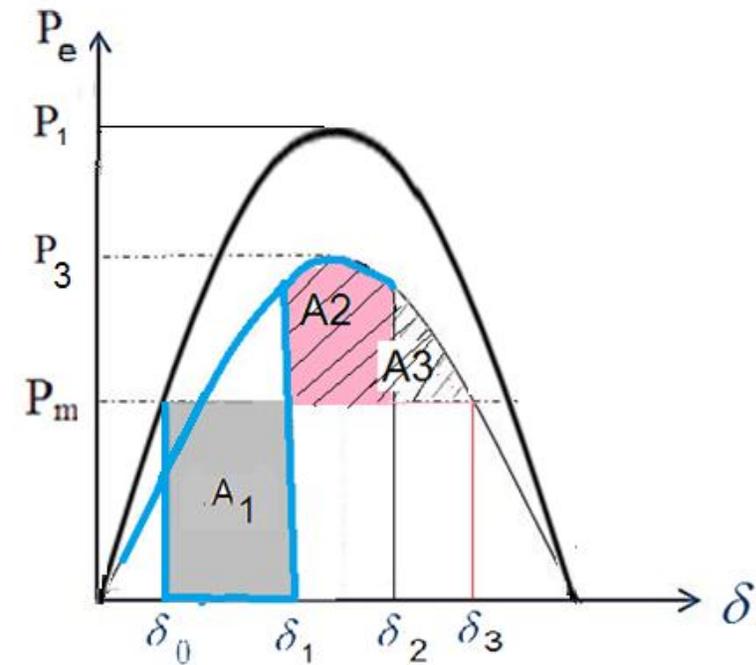


TRANSIENT STABILITY - Equal Area Criteria

The following figures show the trajectory of generator active power and its internal source angle in stable and unstable situation for the above network. The same equations used in pages 22 and 23 can be used for new areas with appropriate selection of δ_0



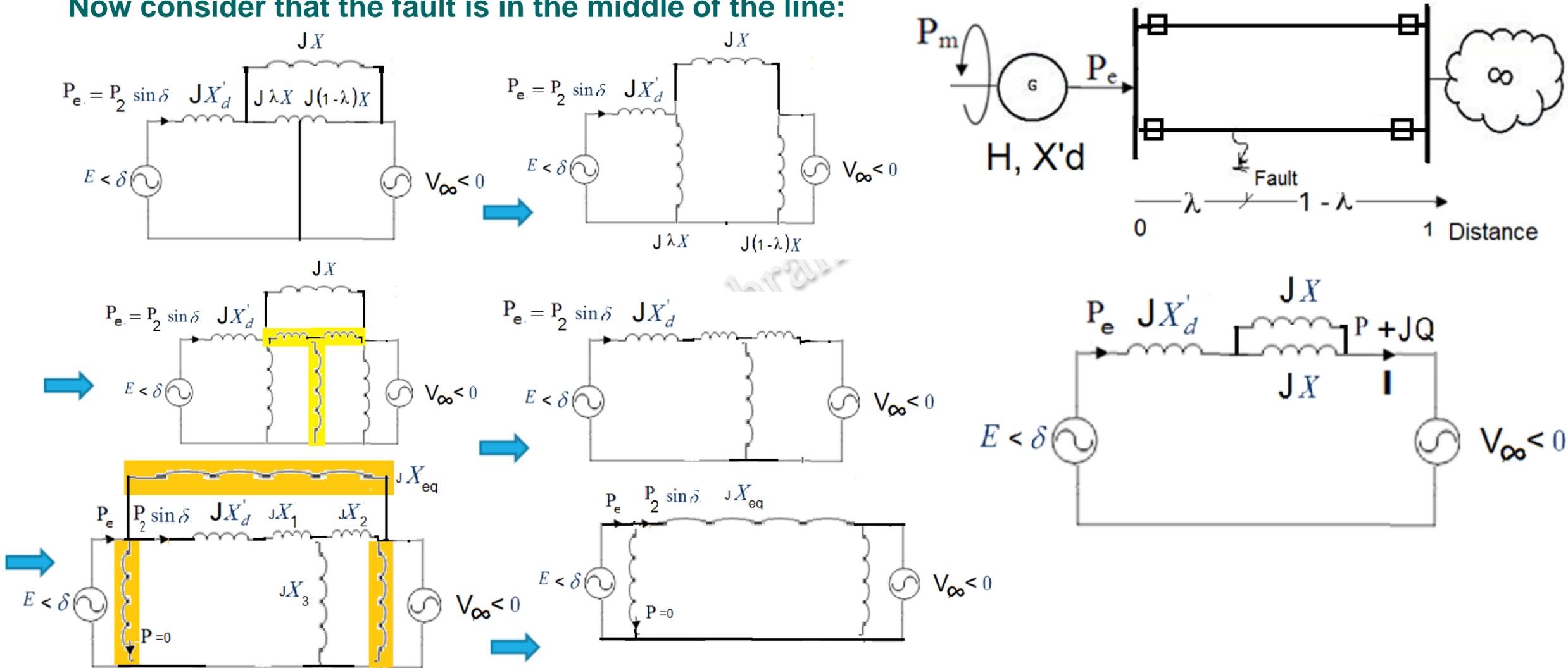
Unstable situation



Stable situation

TRANSIENT STABILITY - Equal Area Criteria

Now consider that the fault is in the middle of the line:



TRANSIENT STABILITY - Equal Area Criteria

You may notice that in this case the power is transmitting during the fault. We need two sources which their magnitude of voltages are fixed to find the amount of power delivery during the fault.

The electrical equivalent circuit can be reduced to the equivalent figure shown.

The students have to find the magnitude of $X_{eq} = ?$ as their assignment and pass it to the TA.

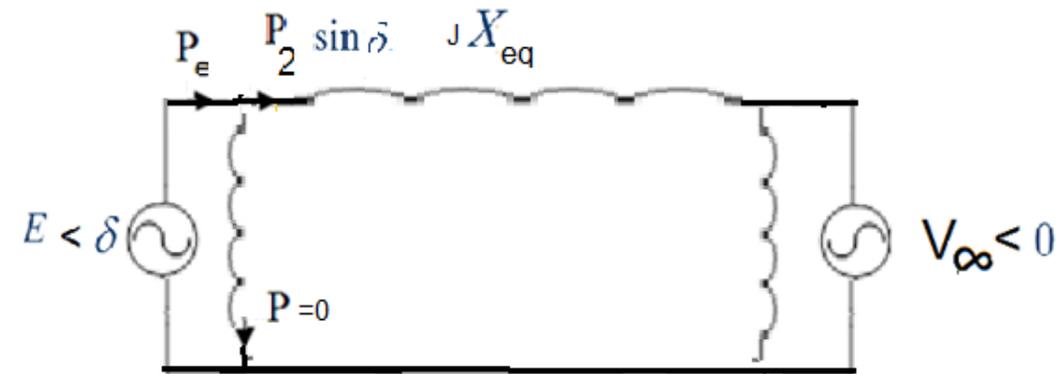
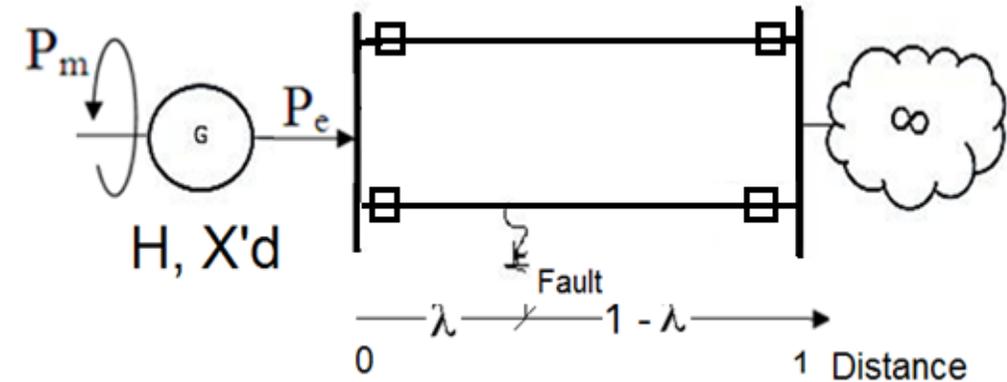
Then P_2 is :

$$P_2 = \frac{E V_\infty}{X_{eq}}$$

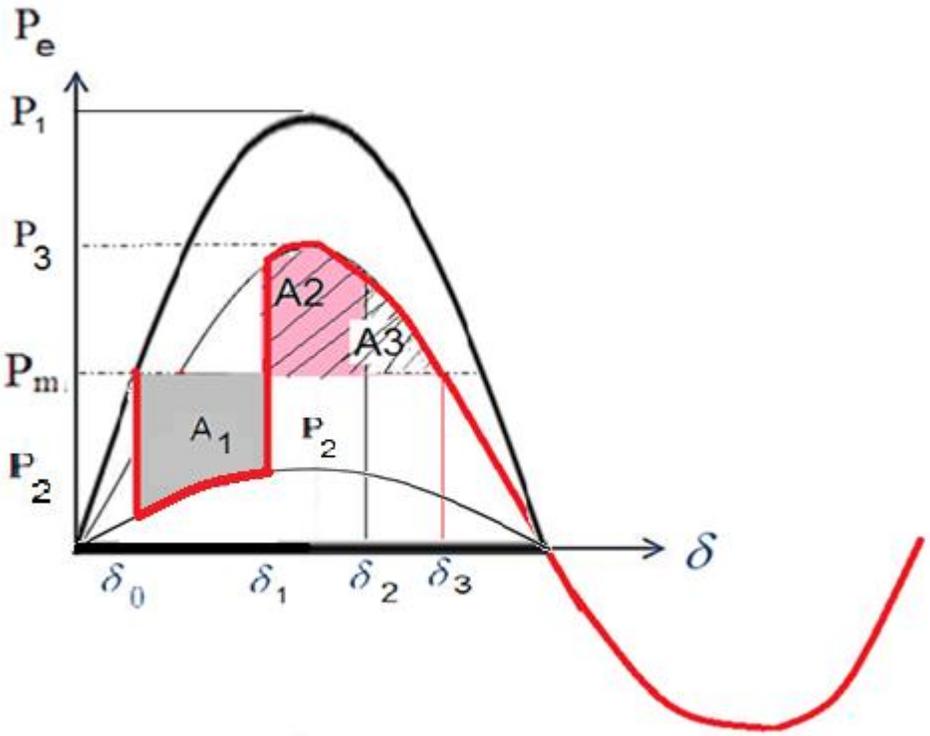
And therefore P_e is:

$$P_e = P_2 \sin \delta$$

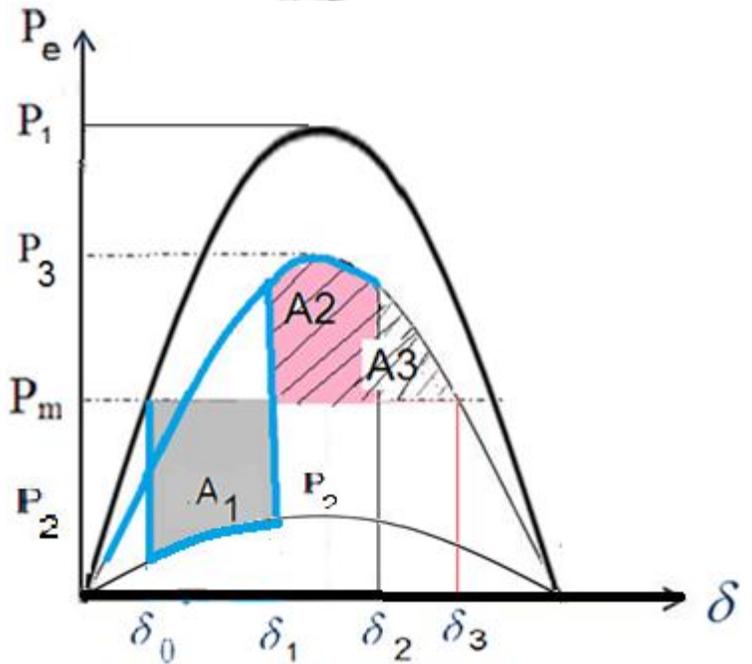
Again the same equations used in pages 22 and 23 can be formulated for the new areas shown in the next page with appropriate selection of δ_0



TRANSIENT STABILITY - Equal Area Criteria



Unstable situation



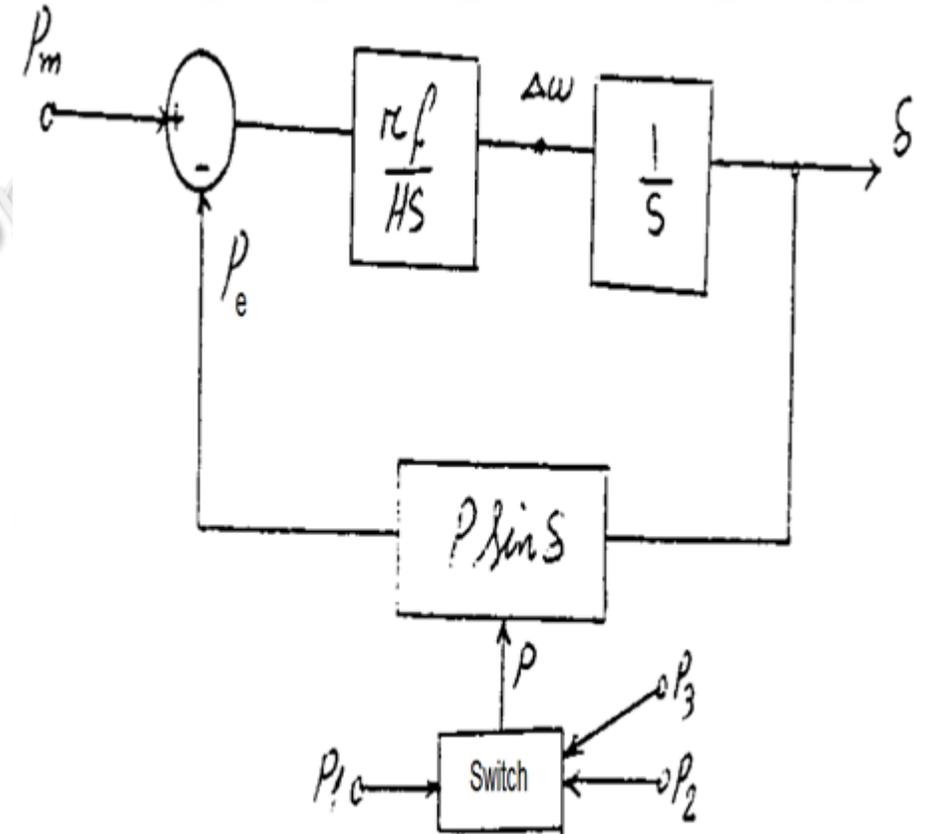
Stable situation

TRANSIENT STABILITY - ANALOG COMPUTER

Analog computer were used to solve the above equations. Here, we need to convert the system equations into block diagram form.

The following figure shows such a block diagram transfer function provided to solve the one machine – infinite busbar transient stability problem.

These days digital computer Programs have a module named User Defined Equipment Modelling (UDEM) which acts like an analog computer.



TRANSIENT STABILITY - DIGITAL COMPUTER

Solution to Swing Equation Step-by-Step Method

Solution to swing equation gives the change in δ with time. Uninhibited increase in the value of δ will cause instability. Hence, it is desired to solve the swing equation to see that the value of δ starts decreasing after an initial period of increase, so that at some later point in time, the machine reaches the stable state. Generally 8, 5, 3 or 2 cycles are the times suggested for circuit breaker interruption after the fault occurs. A variety of numerical step-by-step methods are available for solution to swing equation. The plot of δ versus t in seconds is called the swing curve. The step-by-step method suggested here is suitable for hand calculation for a single machine connected to system. (Obtained from power system analysis P.S.R Murty)

We use rectangular rule to solve our differential equations, although it is proved that the trapezoidal rule is the one most appropriate for power system stability solution method.

TRANSIENT STABILITY - DIGITAL COMPUTER

The step by step method starts from time zero and with the values of the initial condition known and in each time step will proceed by an amount Δt to solve a first order differential equation:

$$\frac{d y}{d t} = f(y,x,t) \quad , \quad y = y_0 \text{ at } t = 0$$

$$y_{n+1} = y_n + [f(y_n, x_n, t_n)] \Delta t$$

Collected by: Prof Shahram Montaseri Kouhsari